A Text Book on

ENGINEERING MECHANICS

for

GATE

PSUs & Other Competitive Exams

Dr. U.C. Jindal

M.Tech, Ph.D.

Former Professor & Head of the Department

Department of Mechanical Engineering

Delhi College of Engineering, Delhi

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Engineering Mechanics for GATE, PSUs and Other Competitive Exams

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Preface

At the outset I must thank Mr. B. Singh (Chairman and Managing Director of MADE EASY Group) who has very kindly given me an opportunity to serve the student community through the publication of this book on Engineering Mechanics.

The present book has been designed as a self study book taking into account the severe shortage of technical teachers in engineering colleges and technical institutions.

The text in the book is well explained through examples supplemented by self explanatory illustrations, exercises supplemented by hints, key points to remember, thought provoking multiple choice questions, special problems so that a student can learn this basic subject in the shortest possible time.

The book covers all the syllabi in Engineering Mechanics of GATE, PSUs, all the universities, IITs, NITs, deemed universities. Students appearing in competitive examinations and other competitive examinations will find the book as an asset to them. The book also serves the purpose of AMIE students.

The book will greatly help the students who could not grasp the subject in the class room.

Any suggestion for the improvement in the text of the book will be thankfully acknowledged.

Dr. U. C. Jindal
Author

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02

CHAPTER

Equivalent Coplanar Force Systems

2.1 Introduction

Several forces in a body can be represented by *an equivalent force* on the same body. Similarly an *equivalent couple* of several couple moments can be determined. In this chapter we will study about following cases:

1. Equivalence of a force at one point represented by a force and a couple at some other point.
2. Resultant of a system of concurrent and coplanar forces.
3. Resultant of a system of coplanar forces and couples.
4. Resultant of parallel forces.
5. Resultant of coplanar distributed forces.

For determining the equivalence of any force system, following basic principles are followed:

(*a*) Sum of a set of concurrent forces in a plane is *a single force* (equivalent to original system of forces)

Three forces F_1, F_2 and F_3 act a point O in x-y coordinates system as shown in Fig. 2.1 (*a*). To find equivalent of these forces, we can use 3 methods:

(*i*) Making a Force Polygon

(*ii*) Taking components of forces in xyz-directions

(*iii*) By addition of force vectors.

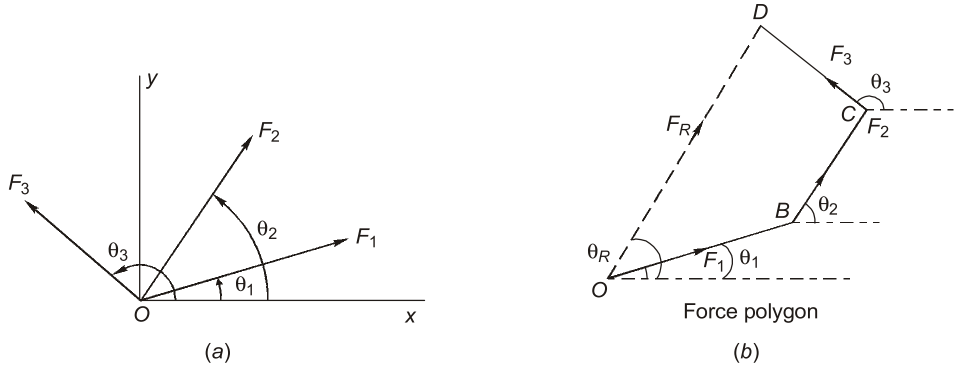

Fig. 2.1

Figure 2.1 (*b*) shows the force polygon. At point O, draw a vector OB equal in magnitude and direction of force F_1, then from point B, draw vector BC parallel to force F_2 and equal in magnitude of F_2. From C draw a line CD parallel to force F_3 and equal in magnitude of F_3. Join OD, from O to D, it is a *resultant force*, F_R = resultant of forces F_1, F_2 and F_3.

- The simplest resultant of a parallel force system is either a force or a couple.
- Load distributed along the length of a beam, such that rate of loading $w_x = w(x)$, a function of x.

  Resultant force, $F_R = -\int w(x)\,dx$

  Point of application of F_R, $\bar{x} = \dfrac{-\int w(x)\,x\,dx}{-\int w(x)\,dx}$.

PRACTICE PROBLEMS

2.1 A force of 200 N is applied at *A* on rigid body *ABC* as shown in **Fig. 2.21**. Replace this force by equivalent couple and force at *C*.

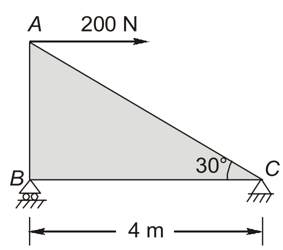

Fig. 2.21

[**Ans:** At *C*, 200 N force in the same direction –461.8 Nm (*cw*)].

2.2 A force of 600 N is applied at *A* (0, 6) m of a rigid body. What is the equivalent force and couple at point *B* (7, 0) m as shown in **Fig. 2.22**?

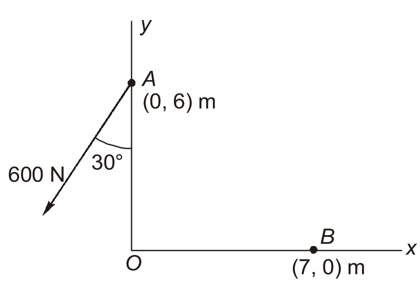

Fig. 2.22

[**Ans:** Force P=–300*i* – 519.6*j* N; Couple=+5437.2*k* Nm].

2.3 A beam *ABCDEF*, 6 m long with vertical projection *DE* = 1 m, is subjected to forces and couples as shown in the **Fig. 2.23**. Determine the simplest resultant of this force and couple system.

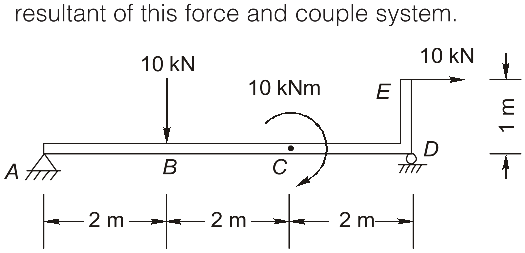

Fig. 2.23

[**Hint:** Moment at *D* is 10 kNm (*cw*)]

[**Ans:** 10 kN at point *C*].

2.4 Determine the resultant of 3 forces acting at *B*, *F* and *D* and a couple at *E* as shown in **Fig. 2.24**. Determine the resultant and the point of application of resultant.

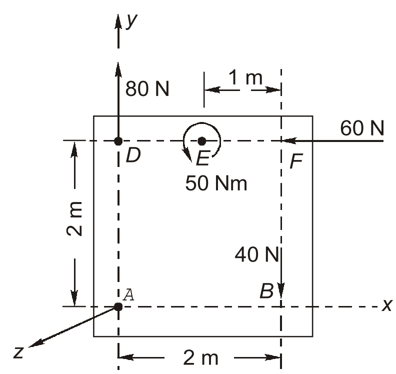

Fig. 2.24

[**Ans:** $|F_R| = 72.1$ N, $F_{Rx} = -60$ N, $F_{Ry} = +40$N, $\bar{x} = -2.25$ m, $\bar{y} = -1.5$ m].

2.5 A beam *ABC*, 5 m long carries distributed load as shown in **Fig. 2.25**. Maximum rate of loading is 4 kN/m at end *C*. There is linear variation of intensity of loading from *A* to *B*. Determine simplest resultant of forces and line of action of simplest resultant from end *A*.

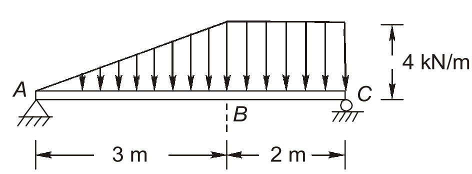

Fig. 2.25

[**Ans:** $F_R = 14$ kN, $\bar{x} = 3.143$ m].

2.6 A cantilever 7 m long carries distributed loads as shown in **Fig. 2.26**, constant rate from *A* to *B*, equates to 8 kN/m and linearly varying load from *B* to *C* with $w = 6$ kN/m at *B* and $w = 0$ at *C* in the upward direction.

Determine simplest resultant of forces and its line of action.

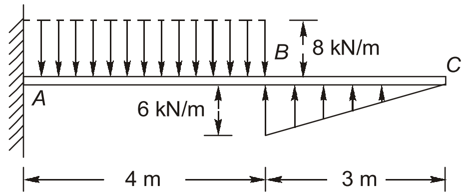

Fig. 2.26

[**Ans:** 23 kN, $\bar{x} = 0.826$ m from *A*].

MULTIPLE CHOICE QUESTIONS

2.1 In *x-y* plane two forces $F_1 = 40i - 30j$ N and $F_2 = -30i + 50j$ N are acting. What is the resultant force (forces are passing through the origin)?

(*a*) $10i + 20j$ N (*b*) $70i + 80j$ N

(*c*) $40i + 20j$ N (d) None of these.

2.2 In *x-y* plane, forces $F_1 = 40i + 50j$ and $F_2 = -30i - 10j$ are passing through the origin. What is the angle of inclination of resultant force with *x*-axis?

(*a*) 14° (*b*) 35°

(*c*) 76° (*d*) 90°.

2.3 Two vertical forces $F_1 = 300$ N↓ and $F_2 = 100$↑ are acting at points (1, 0, 3) and (2, 0, –2) m respectively. What is resulting force, at which points it is acting?

(*a*) 200 N, (2.5, 0, 3.5)

(*b*) 200 N (0, 2.5, –3.5)

(*c*) 300 N (2.5, 0, 3.5)

(*d*) None of these.

2.4 A beam *ABCD*, 6 m long carries a linearly variable load as shown in **Fig. 2.27**. What is the equivalent load and its point of applications from *A*?

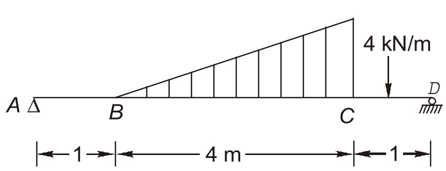

Fig. 2.27

(*a*) 8 kN, 3 m from *A*

(*b*) 8 kN, 3.67 m from *A*

(*c*) 8 kN, 4 m from *A*

(*d*) 8 kN, 5 m from *B*

2.5 A beam 10 m long carries loads and a moment as shown in **Fig. 2.28**. What is equivalent load and point of its application from *A*?

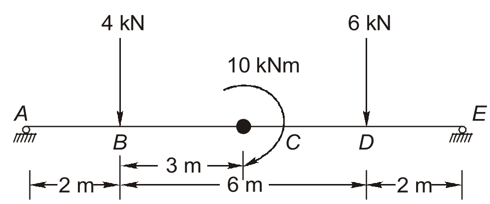

Fig. 2.28

(*a*) 20 kN, 6.6 m from *A*

(*b*) 10 kN, 6.6 m from *A*

(*c*) 0 kN, 5 m from *A*

(*d*) None of these.

2.6 Three forces activity at a point *O* are

$P_1 = (3i + 6j)$ N, $P_2 = (-1.5i + 4.5j)$ N and

$P_3 = (-10.5i + 1.5j)$ N

If a fourth force P_4 is added such that the point *O* is in equilibrium, then force P_4 will be

(*a*) $(15i - 15j)$ N (*b*) $(9i - 12j)$ N

(*c*) $(-9i + 12j)$ N (*d*) $(15i + 15j)$ N

2.7 Figure shown the force acting as the body. Each square is 3 cm × 3 cm. If

$F_1 = 30$ N, $F_2 = 15$ N, $F_3 = 25$ N, $F_4 = 20$ N

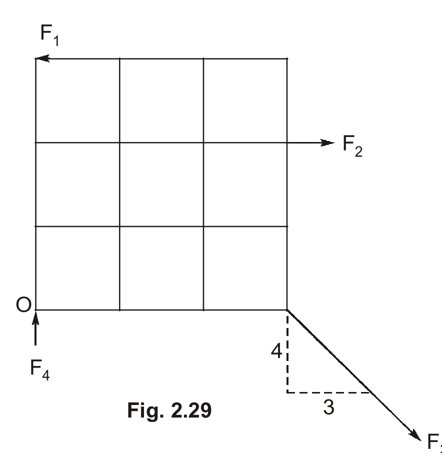

Fig. 2.29

06

CHAPTER

Friction

6.1 Introduction

Friction is a necessary evil. Many actions in daily life can not be performed without the aid of frictional force. A man cannot walk on road, if there is no friction between the feet of the man and the road. Friction takes place between two contacting surfaces.

- Contacting surfaces between two bodies can support normal as well as tangential forces.
- When one surface of a body tends to slide over another surface, frictional force (a tangential force) opposes the motion.
- The contacting surfaces resist the applied tangential force by frictional resistance between the surfaces.
- As the applied force increases, the frictional resistance also increases but to a limit.
- However, when the applied force becomes greater than the limiting value of frictional force, one body slides over another body.
- In a machine, there is loss of energy due to friction and efficiency of the machine is reduced.
- Due to friction, heat is generated between two surfaces, and this heat is dissipated away by lubrication.
- However, frictional force is used to advantage in friction drive as belt, rope, clutch drive.
- In the case of wheeled vehicles, friction is necessary for starting, moving and stopping the vehicle.

Block A of weight W rests on another block B as shown in Fig. 6.1. A force P is applied on block A and frictional force F is generated between the two contacting surfaces as shown. If the block A does not move, then $P = F$, Normal reaction from block B on block A is N, such that $W = N$. Forces P and F constitute a couple (*cw*) and to balance this W and N produce a couple (*ccw*) as shown. However in making calculations for frictional force and normal reactions, the effect of these couples is neglected and normal reaction is shown in line with the vertical load.

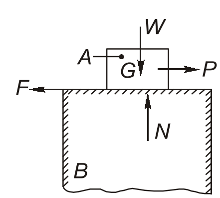

Fig. 6.1

In reality the two contacting surfaces are neither smooth nor plane, they are rough and contain irregularities as ridges and valleys as shown in Fig. 6.2. At contact points, there are frictional forces $F_1, F_2, F_3, ..., F_n$ and normal reactions $N_1, N_2, N_3, ..., N_n$. Resultant of normal reactions and forces of friction *i.e.*, $R_1, R_2, R_3, ..., R_n$ are also shown.

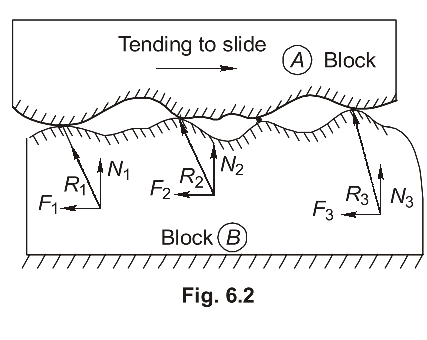

Fig. 6.2

Total normal reaction, $$N = \sum_{i=1}^{n} N_i$$

Total frictional force, $$F = \sum_{i=1}^{n} F_i$$

At the contact points, yielding, crushing and tearing of the material take place and force of friction depends upon:

(*i*) generation of local high temperature
(*ii*) adhesion at contact points
(*iii*) relative hardness of contacting surfaces
(*iv*) presence of thin oxide films, oil, dirt or dust etc. on surfaces.

As the applied force *P* is increased gradually, frictional force also increases gradually but upto a limit F_{max}, beyond which there is no increase in frictional force and body starts slipping *i.e.*, in this example, block *A* starts sliding over block *B*. As the motion starts, there is slight reduction in frictional force and F_{max} is reduced to F_k, kinetic frictional force which remains constant as shown in **Fig. 6.3.**

Friction between two dry or unlubricated surfaces is called ***Coulomb's Friction***.

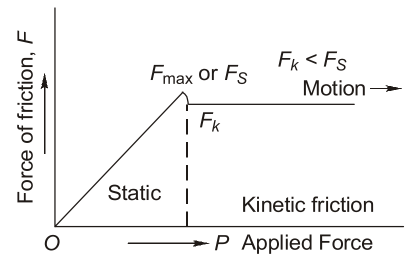

Fig. 6.3

6.2 Laws of Dry Friction or Coulomb's Friction

Frictional force theory or impending slippage or impending motion depends on following laws termed as Coulomb's laws of dry friction, described as follows:

1. **Total frictional force is independent of the magnitude of area of contact between two surfaces**.
2. **Frictional force always opposes the motion and acts in a direction opposite to direction of slippage**.
3. For low sliding velocities, frictional force is practically independent of the magnitude of velocity, however frictional force developed at the instant of impending sliding motion is slightly reduced, and when motion has already started $F_k < F_s$; kinetic frictional force is less than static force of friction.
4. **Total frictional force developed is proportional to the normal reaction provided by mating surfaces**.

Mathematically static frictional force, $F_s \propto N$ (normal reaction)

Kinetic frictional force, $F_k \propto N$

or $F_s = \mu_s \times N$

$F_k = \mu_k \times N,$

where μ_s and μ_k are coefficients of static and kinetic friction respectively between two contacting surfaces. Table 6.1 gives values of μ_s for contacting surfaces.

Table 6.1 Approximate Values of μ_s

Contacting surface materials	μ_s	*Contacting surface materials*	μ_s
Metal on metal	0.15–0.60	Wood on leather	0.25–0.50
Metal on wood	0.20–0.60	Stone on stone	0.4–0.70
Metal on stone	0.30–0.70	Earth on earth	0.2–1.0
Metal on leather	0.30–0.60	Rubber on concrete	0.60–0.95
Wood on wood	0.25–0.50	Rope on wood	0.50–0.80

Corresponding values of μ_k would be about 20–25 per cent smaller than μ_s

6.3 Angle of Friction

Consider a block of weight *W* resting on a horizontal plane as shown in **Fig. 6.4**. Weight acts through centre of gravity, *G* of the block. Now a horizontal force *P* is applied on block and a horizontal frictional force *F* is developed

08

CHAPTER

Second Moment of Area (Moment of Inertia)

8.1 Introduction

A quantitative measure of the resistance of a beam is its *second moment of area* or so called *moment of inertia*.

In strength of materials, the expressions for stresses, slopes and deflections in beams and columns require the use of a term known as *Moment of inertia*, which is equal to $I = \int r^2 dA$, *i.e.*, an area *A*, is divided into small parts such as *dA* and **each small area *dA* is multiplied by the square of its *moments arm r*, about the reference axes**.

Let us consider an area *A* as shown in **Fig. 8.1**. This area can be divided into small parts such as *dA* shown in the figure. Let us consider *ox* as the reference axis.

First moment of this area about *ox* axis = $y \cdot dA$.

Second moment of this area about *ox*-axis = $y^2\, dA$.

Then the mathematical expression $\int y^2 dA$ is known as the moment of inertia of the section about axis *ox*

or $$I_{xx} = \int y^2 dA \text{ (about } x\text{-}x \text{ axis)}$$

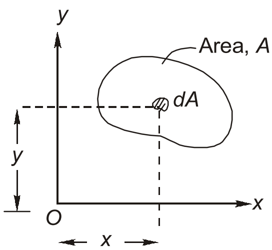

Fig. 8.1

Similarly choosing *oy*-axis as the reference axis, moment of inertia or second moment of area

$$I_{yy} = \int x^2 dA,$$

where *x* is the perpendicular distance of *dA* from the axis *oy*. **The ratio of moment of inertia *I* and area *A* has dimensions of the length to the second power**. This length is called the *radius of gyration of the plane section A*, with respect to the axis of reference. Therefore,

Radius of gyration about *ox*-axis, $$\rho_x = \sqrt{\frac{I_{xx}}{A}}$$

Radius of gyration about *oy*-axis, $$\rho_y = \sqrt{\frac{I_{yy}}{A}}.$$

8.2 Rectangular Section

Consider a rectangular section of breadth *b* and depth *d* as shown in **Fig. 8.2.** Centroid of the section is located at *G*, at distances of *b*/2 and *d*/2 from vertical and horizontal axes *OY'* and *OX'*. Say the reference axis *GX* and *GY* pass through the centroid *G* of the section. Now take a small area *dA* = *b.dy*, of breadth *b* and thickness *dy*.

(a) $\frac{bh^3}{12}$ (b) $\frac{bh^3}{6}$

(c) $\frac{bh^3}{3}$ (d) $\frac{bh^3}{4}$

[CSE, Prelim, CE : 2009]

Answers

8.1 (b)	8.2 (a)	8.3 (c)	8.4 (b)	8.5 (c)
8.6 (b)	8.7 (c)	8.8 (a)	8.9 (d)	8.10 (d)

EXPLANATIONS

8.1 (b)

For equilateral triangle, $I_{xx} = I_{yy}$ along centroidal axes.

8.2 (a)

$$I_{xx} = \frac{bd^3}{12} = \frac{b \times 8b^3}{12} = \frac{2}{3}b^4 = 1000$$

$b = 6.22$ cm.

8.3 (c)

$$I_{xx} = \frac{bh^3}{36}, I_{ab} = \frac{bh^3}{12} = 3I_{xx}.$$

8.4 (b)

$$I_{AB} = \frac{bh^3}{3} = \frac{5(0.866 \times 6)^3}{3} = 233.08 \text{ cm}^4.$$

8.5 (c)

$$I = \frac{\pi(D^4 - d^4)}{64} = \frac{\pi}{64}(10^4 - 6^4) = 427.26 \text{ cm}^4.$$

8.6 (b)

$$I_{xx} = \frac{\pi R^4}{16} - \frac{\pi}{8}\left(\frac{R}{2}\right)^4 = \pi R^4\left(\frac{1}{16} - \frac{1}{128}\right)$$

$$= \pi R^4 \times \frac{7}{128}.$$

8.7 (c)

$$\text{Radius of circle} = \frac{0.866}{3}a = 0.288666a$$

$$I_{xx} = 0.108a^4 - \frac{\pi}{4}(.288666a)^4 = .1025a^4$$

8.8 (a)

$$I_{AB} = \frac{bh^3}{12}, I_{EF} = \frac{bh^3}{36} + \frac{bh}{2}\left(\frac{4h^2}{9}\right) = \frac{bh^3}{36} + \frac{2bh^3}{9} = \frac{bh^3}{4}.$$

8.9 (d)

$I_{xx} = I_{xx} + 1 \times 4 = 10$

$I_{xx} = 6\ m^4$

$P_{yy} = 6 + 1 \times 3^2 = 15 \text{ m}^4$

8.10 (d)

$$I_{xx} = \frac{bh^3}{36} + \frac{bh}{2}\left(\frac{2b}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{2bh^3}{9} = \frac{bh^3}{4}$$

■■■■

14
CHAPTER

Kinetics of a Particle

14.1 Introduction

In Kinetics, we study about the magnitude and direction of forces involved in the motion of a particle or a body. Newton's Second Law of motion states that a force is necessary for the change of state of rest or of uniform motion of a body and in many engineering problems, Newton's Second Law is directly applied for the study of acceleration of a body. There are various types of accelerations such as linear acceleration, tangential acceleration, normal acceleration, angular acceleration of a body in motion of translation and or in motion of rotation.

The bodies possess definite shape and size. However when the *motion of all the particles of a body is defined by same parameter then the body can be simplified as a particle*. Such as a coach of a train, when all the particles of the coach possess same velocity and same acceleration, then it can be classified simply as a *particle*. Similarly the motion of a rocket or satellite can be analysed considering them to be particles of given mass. While analysing the effect of a force during any motion, the particle usually selected as the one located at the mass centre of the body and the resultant of all the forces acting on the translating body passes through its mass centre.

14.2 Newton's Second Law of Motion

Several forces F_1, F_2, F_3, ..., F_n are acting on a body as shown in Fig. 14.1. If the resultant of these forces is not zero, then body will experience acceleration in the direction of the resultant force

$$F_R = \sum_{i=1}^{n} F_i \neq 0$$

and the body/or the particle will move in the direction of resultant force such that $F_R = m \cdot a_R$, where a_R is acceleration in the direction of F_R.

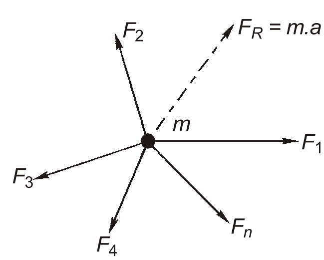

Fig. 14.1

In a three dimensional case, resultant force can be expressed as

$$\sum F = F_R = \left(\sum F_x\right) i + \left(\sum F_y\right) j + \left(\sum F_z\right) k$$
$$= F_{xR} i + F_{yR} j + F_{zR} k$$

Similarly acceleration $\quad a_R = a_x i + a_y j + a_z k$

where a_x, a_y, a_z are components of acceleration along 3 co-ordinates axes.

Now let us consider that a particle is subjected to forces F_1, F_2, F_3 and particle is subjected to accelerations a_1, a_2 and a_3 respectively, these acceleration components are proportional to forces applied *i.e.*,

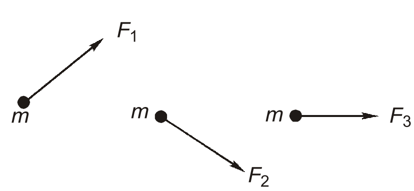

Fig. 14.2

i.e., $a_1 \propto F_1,\ a_2 \propto F_2;\ a_3 \propto F_3$

or $$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = a \text{ constant } = m \text{ (mass)}$$

So m is a measure of some property of a particle that does not change and remains constant. *It is a measure of inertia of a particle*. More is the mass, more is the inertia of the particle to change its state of rest or of uniform motion.

Equation of motion subjected to external forces $\sum F = m\,.\,a$

$$\sum (F_x i + F_y j + F_z k) = m\,(a_x i + a_y j + a_z k).$$

14.2.1 Reference System

During the analyses of motion of a body or particle, system of reference cannot be arbitrary. These co-ordinate axes must have constant orientation with respect to space. Therefore the origin of the co-ordinate system should be attached to the sun, which is the mass center of solar system. Such a system of axes is known as *Newtonian frame of reference*. A system of axes attached to the earth does not constitute a Newtonian frame of reference, because the earth rotates with respect to the space and is accelerated with respect to the sun. However no appreciable error is caused in the solution of common engineering problems if acceleration of a body is determined with respect to the axes attached to the earth.

14.2.2 System of SI Units

In SI units, a Newton is a force acting on a body of mass 1 kg producing an acceleration of 1 m/s^2 in the body as shown in **Fig. 14.3**.

In a *vertical direction*, a body is attracted by the earth towards its center with acceleration $g = 9.81$ m/s^2 (on the surface of the earth). Force on body of mass 1 kg is attracted by a force of 9.81 Newton. This *force of attraction* by earth is known as *weight of the body*.

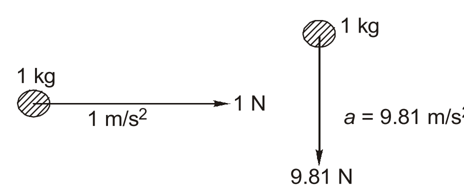

Fig. 14.3

14.3 Types of Problems

As per Newton's second law, $\sum F = ma$, if acceleration is specified then force required by a body of mass m is determined. If force is specified then acceleration can be determined. However if the force is a function of time, position, velocity or acceleration, then equation $\sum F = ma$, becomes a *differential equation*.

Example 14.1 At a certain instant a body of mass 18 kg falling freely under gravity was found to be falling at the speed of 28 m/s. What force will stop the body in (*a*) 3 seconds, (*b*) 30 metres (**Fig. 14.4**)?

Solution Mass, $m = 18$ kg

Speed $V = 28$ m/s

$g = 9.81$ m/s^2

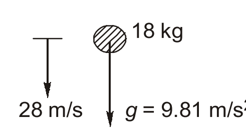

Fig. 14.4

Body is falling under gravity with initial speed of 28 m/s.

(*a*) **Body is to be stopped in 3 seconds:**

Say a' is the retardation.

After three seconds, velocity $= 28 + 3 \times 9.81 = 57.43$ m/s

Retardation, $$a' = \frac{57.43}{3} = 19.143 \text{ m/s}^2$$

Force required, $$F = ma' = 18 \times 19.143 = 344.57 \text{ N}$$

20

CHAPTER

Impulse and Momentum

20.1 Introduction

Newton's second law of motion states that the externally applied force is responsible for the rate of change of momentum in a body. In other words this law gives relationship between force, mass and acceleration *i.e.*, rate of change of velocity. The principle of impulse and momentum is derived again from the Newton's second law of motion. This principle is generally used in problems of rigid body dynamics where large forces act for a short interval of time producing change of momentum of the body, such as the forces acting during hammer blow action in smithy work, press work, driving of piles into the ground, firing of shell from a gun etc. The most important examples of industrial use is (*a*) Impulse turbine, in which principle of impulse-momentum is used for power generation. (*b*) Hammer blows—used in forging operation and making connecting rods, crank shafts and many other engineering components.

As per Newton's second law, force = mass × acceleration

or
$$F = m\frac{dV}{dt}$$

or
$$F\,dt = m\,dV$$

Integrating both the sides
$$\int_{t_1}^{t_2} F\,dt = m\int_{V_1}^{V_2} dV$$

or
$$F(t_2 - t_1) = m(V_2 - V_1)$$

Linear impulse = Linear momentum, a large force applied momentarily

When no external force is acting on the system, the principle of impulse-momentum is reduced to principle of conservation of momentum.

Two masses m_1 and m_2 collide with each other with velocities U_1 and U_2 respectively and after impact they separate with velocity V_1 and V_2 respectively as shown in **Fig. 20.1.** Then as per principle of conservation of momentum

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$$

Initial momentum before impact = Final momentum after impact.

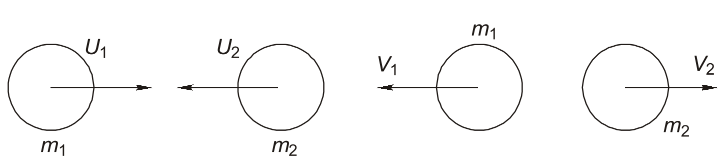

Fig. 20.1

MULTIPLE CHOICE QUESTIONS

20.1 A trolley of 600 kg can move along a horizontal frictionless track, as shown in **Fig. 20.10**. Initially the trolley with a man on it of mass 70 kg is moving towards right at a speed of 5 m/s. If the man starts walking on the trolley with a speed of 2 m/s towards left, what is velocity of travel?

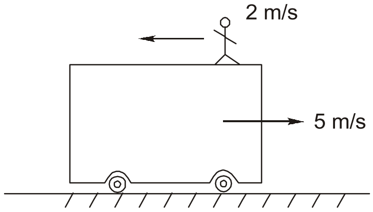

Fig. 20.10

(*a*) 4.79 m/s (*b*) 5.21 m/s
(*c*) 5.81 m/s (*d*) None of these.

20.2 A 32 gram bullet is fired with a velocity of 500 m/s into a wooden block which rests against a rigid vertical wall. The bullet is brought to rest in 0.1 s. What is average impulsive force exerted by the bullet on the block?
(*a*) 160 N (*b*) 1600 N
(*c*) 16 kN (*d*) None of these

20.3 A pile hammer of weight 12 kN strikes on pile of weight 4 kN with a velocity of 3 m/s. How much does a single blow of hammer drive the pile, if the resistance of ground is 200 kN, assume ground resistance to much the uniform
(*a*) 0.12 (*b*) 0.15 m
(*c*) 0.18 m (*d*) 0.36 m

20.4 During service, a tennis player hits the ball when it is on the top of its trajectory (when thrown by hand). The speed of the ball often being hit is 150 km/hour. If the contact of the ball with the racket is for 0.02 s, and weight of ball is 50 gm, what is average force applied by the player on the ball?
(*a*) 69.4 N (*b*) 104.16 N
(*c*) 10.41 N (*d*) None of these

20.5 A 5 kg homogeneous rotor with radius of gyration 0.4 m comes to rest in 60 seconds from a speed of 200 rpm, what was the frictional torque that stopped the rotor?
(*a*) 0.28 Nm (*b*) 0.56 Nm
(*c*) 0.7 Nm (*d*) None of these

20.6 A single degree of freedom system having mass 1 kg and stiffness 10 kN/m initially at rest is subjected to an impulse force of magnitude 5 kN for 10^{-4} seconds. The amplitude in mm of the resulting free vibration is
(*a*) 0.5 (*b*) 1.0
(*c*) 5.0 (*d*) 10

20.7 A player catcher a cricket ball of mass 0.1 kg moving with a speed of 20 m/s If the ball is in constant with his hand for 0.1 s, what is the impulse (approximate) exerted by the ball on the hand of the player?
(*a*) 2 Ns (*b*) 5.2 Ns
(*c*) 10.8 Ns (*d*) 12.5 Ns
[CSE, Prelim, CE : 2007]

20.8 A cricket ball of mass 150 gram moving with a velocity of 12 m/s is hit by a bat so that the ball is turned back with a velocity of 20 m/s. The force of blow acts for 0.01 s on the ball. What is the average force exerted by the bat on the ball?
(*a*) 480 N (*b*) 48 N
(*c*) 248 N (*d*) 48 N
[CSE, Prelim, CE : 2009]

20.9 Which one of the following is stated by the moment of momentum principle of a rotating system
(*a*) angular momentum is conserved
(*b*) vector sum of all external forces acting on a control volume in a fluid flow equals rate of change of linear momentum
(*c*) the resultant force exerted on a body is equal to the rate of change of angular momentum
(*d*) the torque due to resultant force is equal to the rate of change of angular momentum
[CSE, Prelim, CE : 2002]

20.10 A 100 kg flywheel having a radius of gyration of 1 m is rotating at 100 rpm. What is its angular momentum (approximate) of the flywheel about its axes of rotation is kgm^2/s
(*a*) 10000 (*b*) 10470
(*c*) 1000 (*d*) 12000
[CSE, Prelim, CE : 2007]

20.11 A bullet of mass 'm' travels at a very high velocity v (as shown in the figure) and gets embedded inside the block of mass "M" initially at rest on a rough horizontal floor. The block with the bullet is seen to move a distance "s" along the floor. Assuming μ to be the coefficient of kinetic friction between the block and the floor and "g" the acceleration due to gravity what is the velocity v of the bullet?

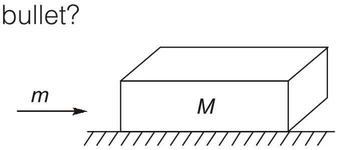

Fig. 20.11

(a) $\frac{M+m}{m}\sqrt{2\mu gs}$ (b) $\frac{M-m}{m}\sqrt{2\mu gs}$

(c) $\frac{\mu(M+m)}{mm}\sqrt{2\mu gs}$ (d) $\frac{M}{m}\sqrt{2\mu gs}$

[GATE 2003 : 1 Mark]

Answers

20.1 (b)	20.2 (a)	20.3 (c)	20.4 (b)	20.5 (a)
20.6 (c)	20.7 (a)	20.8 (a)	20.9 (d)	20.10 (b)
20.11 (a)				

EXPLANATIONS

20.1 (b)

$$(600 + 70)\,5 = (600 + 70 - 70 \times 2)\,V'$$

$$V' = \frac{3490}{670} = 5.21 \text{ m/s.}$$

20.2 (a)

$$F = \frac{mV}{dt} = \frac{0.032 \times 500}{0.1} = 160 \text{ N.}$$

20.3 (c)

$$V = \frac{MV_0}{M+m} = \frac{12 \times 3}{12+6} = 2 \text{ m/s}$$

$$\frac{1}{2}(M+m)V^2 = R.S$$

$$\frac{1}{2}(18) \times 2^2 = 200 \times S$$

$$36 = 200\text{ s}$$

$$S = 0.18 \text{ m.}$$

20.4 (b)

$$F = \frac{mV}{t} = 0.050 \times \frac{150{,}000}{3600} \times \frac{1}{0.02} = 104.16 \text{ N.}$$

20.5 (a)

$$\text{Torque} = I\alpha = 5 \times 0.4^2 \times \left(\frac{2\pi \times 200}{60 \times 60}\right) = 0.28 \text{ Nm.}$$

20.6 (c)

Impulse= $5000 \times 10^{-4} = 0.5$ Ns $= m.A.\omega$,

where $\omega = \sqrt{k/m}$, $= 100$ rad/s^2

A = amplitude, m = mass

$0.5 = 1 \times 100 \times A$, $A = .005$ m $= 5$ mm

20.7 (a)

$$v = 20 \text{ m/s}$$

$$m = 0.1 \text{ kg}$$

$$\text{Impulse} = 20 \times 0.1 = 2 \text{ Ns}$$

20.8 (a)

$$m = 0.15 \text{ kg}$$

$$v - u = 12 - (-20) = 32 \text{ m/s}$$

$$\text{Average force} = \frac{mV}{dt} = \frac{0.15 \times 32}{0.01} \text{ Ns}$$

$$= 480 \text{ Ns}$$

20.9 (d)

$$\text{Angular momentum} = I\omega$$

$$\text{Torque, } T = \frac{A}{dt}(I\omega)$$

$$= I - \frac{d\omega}{dt}$$

$$= I\alpha$$

= rate of change of angular momentum

20.10 (b)

$$I = 100 \times 1^2 = 100 \text{ kg m}^2$$

$$\omega = \frac{2\pi \times 1000}{60} = 104.72 \text{ rad/sec}$$

$$I_\omega = I\omega = 100 \times 104.72$$

$$= 10472 \text{ kgm}^2\text{/s}$$

20.11 (a)

$$mV = (m + M)V'$$

$$V'^2 = \left(\frac{mV}{m+M}\right)^2$$

$$0 = V'^2 = 2\mu gs = \left(\frac{mV}{m+M}\right)^2 - 2\mu gs$$

$$\left(\frac{mV}{m+M}\right)^2 = 2\mu gs$$

$$V = \frac{M+m}{m}\sqrt{2\mu gs}$$

■■■■